FINITE ELEMENT ANALYSIS OF NATURAL RUBBER ISOLATOR

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Summary

In this study, the buckling of natural rubber bearings was examined by finite element analysis. It examined how the material properties of the rubber (nonlinear properties) or the thickness of the interlayer steel plates affected the buckling load. The bearings were analyzed by applying loads in two ways: applying a horizontal deformation on the bearings under a constant compressive load, and applying a compressive load on the bearings under a constant shear strain. From the results, it was found that the buckling load increased as the hardening property of the rubber material was strengthened. It was also found that the resistant to buckling was higher as the interlayer steel plates in the bearings were thicker.

1. Introduction

The ultimate performances of rubber bearings are often defined by the point when buckling occurs. The approximate calculation of the buckling load, σ_{cr}, of rubber bearings is obtained by the following equation.

$$\sigma_{cr} = \zeta G \cdot S_1 \cdot S_2$$

where, G : shear modulus, $S_1 = \frac{D}{4t_R}$: first shape factor, $S_2 = \frac{D}{nt_R}$: second shape factor

ζ : a factor determined by rubber material or the shape of a rubber bearing

D: the diameter of a rubber bearing, t_R : thickness of one rubber layer

n : the number of rubber layers

From the above equation, buckling load is proportional to the shape factors of rubber bearings. A lot of experiments have been conducted in order to determine the compressive load and horizontal deformation when the rubber bearing buckled. The buckling point is defined when the horizontal stiffness (tangential stiffness) is zero on the hysteresis loop obtained from compression shear tests. The rubber bearings are prone to buckling under a smaller compressive load as the horizontal deformation is larger. The buckling point is estimated by the relationship to the compressive load on the bearings and horizontal deformation based on the experimental results.

However, the buckling of rubber bearings is actually affected by the size of the central hole, the thickness of the interlayer steel plates, the material properties of the rubber, the manufacturing errors, and so on. The details of the effects and causes of buckling have not yet to be clarified. This study focuses on the properties of rubber material and the different thicknesses of the interlayer steel plates of the bearings, and clarifies how they affect buckling by finite element analysis.

2. Analytical Model and Analysis Method

2.1 Analytical Model

In the analysis, natural rubber bearings 500mm in diameter were used as the analytical model. The models have 26 rubber layers of 3.75mm thickness (97.5mm of total rubber thickness), and the model do not have central hole. The first shape factor is 33.3 while the second shape factor is 5.1. The basic interlayer steel plate, t_S, is 3.2mm thick. Flange plates, which are 25mm thick and 700mm in diameter, are attached to the top and bottom of the bearings.

The analytical model is shown in Fig. 1. The meshes are radially partitioned into ten and circumferentially partitioned into eight. The elements used are 8-node 3D isoparametric elements. The rubber layers and interlayer steel plates are partitioned into two in the direction of thickness.

Loads were applied to the models in two different ways. One method was uniformly applying the shear strain to the models after applying a certain level of compressive load. The shear deformation was applied every 5mm monotonically. The compressive load was between 15MPa and 80MPa. The other method was that a certain level of horizontal deformation was applied to the models. After that, compressive load was applied to the models in 10MPa increments, keeping the deformed shape. The amount of horizontal deformation was set as 100mm, 200mm, and 300mm, whose respective shear strain were about 100%, 200%, and 300% for each.

The analytical models in this study are shown in Table 1. They are five combinations of three types of rubber material and interlayer steel plates of varying thicknesses. The analytical program MARC2003 was used.

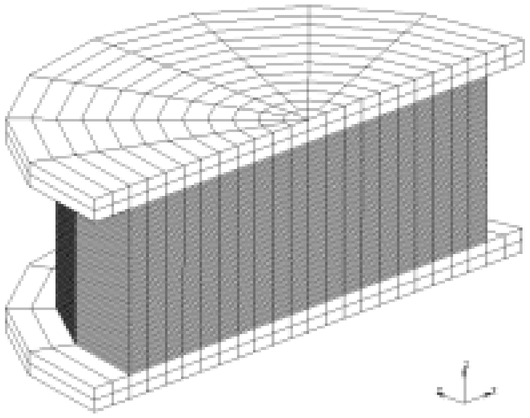


Fig.1 Analytical Model

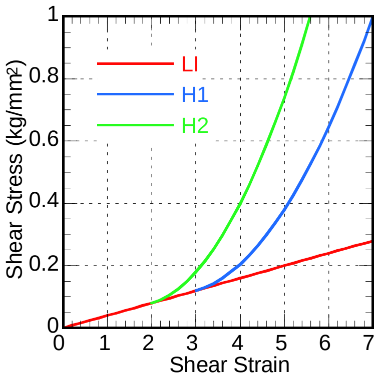


Fig.2 Rubber Material Properties

Table 1 Analytical Models

Rubber Type	Thickness of Shim Steel Plates (ts)		
	2.1mm	3.2mm	4.8mm
LI		X	
H1	X	X	X
H2		X	

2.2 Steel Material Property

The steel material is modeled on elastic-perfectly plastic. As the yield condition, the von Mises criterion is adopted, while the isotropic hardening rule is adopted as the workhardening rule. The Young's modulus of the steel material is 205GPa, the Poisson's ratio was 0.3, and the yield stress is 294MPa (30kg/mm^2). Interlayer steel plates 2.1mm thick and 4.8mm thick are added to those 3.2mm thick in the analysis.

2.3 Rubber Material Property

The rubber material is modeled using the strain energy density function, *W*. In this study, Equation (1) was used. The equation, which allows for the compressibility of the rubber material, was proposed by Ogden.

$$W = \sum_{n=1}^{N} \frac{\mu_n}{\alpha_n}\left[J^{\frac{-\alpha_n}{3}}\left(\lambda_1^{\alpha_n}+\lambda_2^{\alpha_n}+\lambda_3^{\alpha_n}\right)-3 \right] + 4.5K\left(J^{\frac{1}{3}}-1 \right)^2 \qquad (1)$$

where, $J=\lambda_1\lambda_2\lambda_3$, λ_n: principal stretches, μ_n, α_n: the material constants

K: the bulk modulus.

Three types of rubber material with different properties were used as shown in Fig.2. The initial shear modulus of all of the materials was 0.392MPa (4kg/cm^2), though the hardening properties are different in the region where shear strain is more than 200%.

Type LI is linear rubber material while Type H1 and Type H2 show hardening properties. Type H2 shows stronger hardening properties than Type H1. The relationship between shear stress and shear strain in the hardening property can be expressed in a quadratic function, and the shear modulus (tangential stiffness), *G*, is proportional to shear strain, γ. G/γ =0.09 for Type H1 while G/γ =0.12 for Type H2.

Table 2 shows the material constants of the rubber materials, which were obtained by approximate calculation using Equation (1). Using the material constants in Table 2, the shear modulus of the rubber material, *G*, is calculated as follows:

$$G = \frac{1}{2}\sum \alpha_n \mu_n \qquad (2)$$

The shear modulus obtained by Equation (2) is in the range of about 4kg/cm^2±10%. The bulk modulus is set as 5000 times as much as the shear modulus.

Table 2 Material Constants of Rubber （kg/mm^2）

Rubber Type	n	1	2	3	G	K
LI	μ_n	0.04	0.3539	—	0.04 (0.392MPa)	200 (1.96GPa)
	α_n	2	9.348×10^{-16}	—		
H1	μ_n	7.927×10^{-4}	0.04438	0.2489	0.0447 (0.438MPa)	223.5 (2.19GPa)
	α_n	4.625	1.061	0.1554		
H2	μ_n	7.269	3.6576	0.003241	0.0365 (0.358MPa)	182.6 (1.79GPa)
	α_n	0.002692	0.01086	4.2508		

3. Analytical Results

In Fig.3, the shear loading analysis results of rubber bearings under constant compressive loads are shown. From the relationship between the horizontal load and horizontal deformation, it was found that horizontal stiffness and deformation capacity decreased as the compressive load increased.

It can be said that the hysterisis property of the rubber bearings is stable when the horizontal stiffness is positive in the hysterisis loop, allowing it to act as a restoring force. It is evident that rubber bearings become unstable when horizontal stiffness decreases as the compressive load and horizontal deformation increase. The rubber bearings are considered to buckle when horizontal stiffness is negative. We consider that it is the buckling point where the tangential stiffness becomes zero. The buckling points present the combination of compressive load and horizontal deformation.

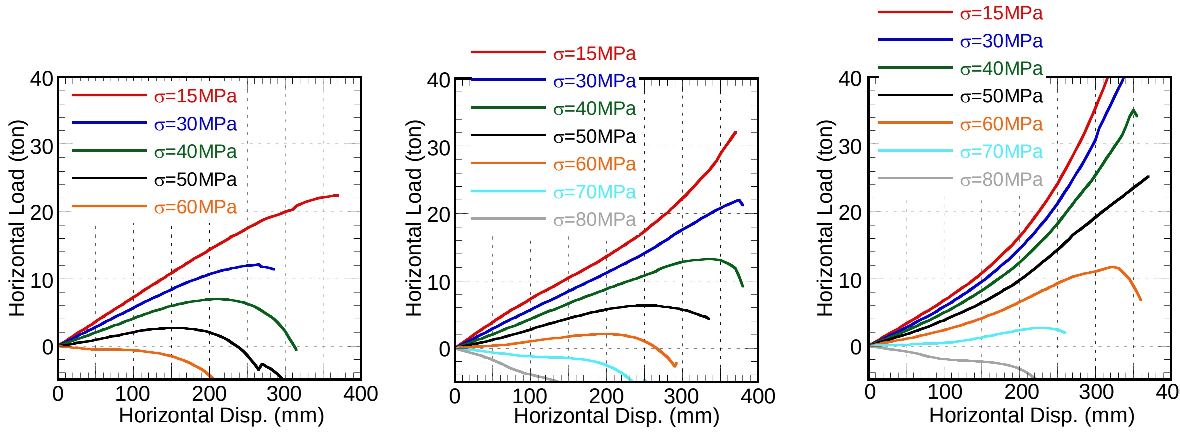
The hysterisis loop of Type H2 rubber material is the most stable and the buckling load is the largest, followed by Type H1, then Type LI. The buckling point is higher as the interlayer steel plate is thicker. Also the compressive load dependence on the horizontal stiffness lowers with increasing the thickness of steel plate.

Fig.4 shows the relationship between horizontal stiffness and compressive stress obtained from the hysterisis loops in Fig.3. The horizontal stiffness is the secant stiffness when horizontal displacement is 50mm. The horizontal stiffness decreases as the compressive load is increased. The compressive load when the horizontal stiffness becomes zero is the buckling load of the rubber bearings, which has been theoretically ascertained. The relation between horizontal stiffness and compressive load is calculated approximately by Equation (3) as follows:

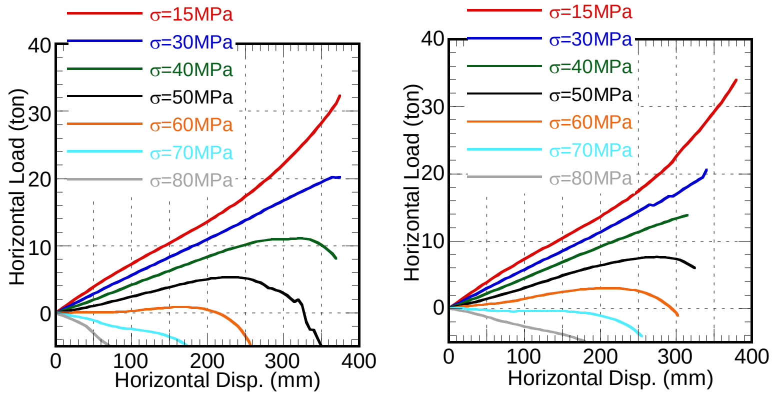
$$\frac{K_H}{K_{H0}} = 1 - \left(\frac{\sigma}{\sigma_{cr}}\right)^2 \qquad (3)$$

where, K_{H0} : horizontal stiffness when there is no compressive load

K_H : horizontal stiffness, σ : compressive load, σ_{cr} : buckling load



(a) ts=3.2mm, LI-type (b) ts=3.2mm, H1-type (c) ts=3.2mm, H2-type



(d) ts=2.1mm, H1-type (e) ts=4.8mm, H1-type

Fig.3 Horizontal Characteristics of Shear Loading under Constant Compression Load

In Fig.4, a prediction equation is written. Also, Equation (3) almost corresponds to the analytical results. In the analysis, the horizontal stiffness decreases as the compressive load is increased. However, the buckling load is different according to the rubber material or the thickness of the interlayer steel plate. The buckling load of Type LI is the smallest while the buckling load of Type H2 is the largest. The buckling load of Type H2 is about 30% larger than Type LI. Also, the buckling load tended to be larger when the interlayer steel plate was thicker.

Fig.5 (a) and (b) show the analytical results for rubber bearings of Type H2 with interlayer plates 3.2mm thick. Fig.5 (a) shows the relationship between compressive stress and vertical displacement when a compressive load is applied to the bearings while its horizontal deformation is sustained. On the other hand, Fig.5 (b) shows the relationship between compressive load and shear force (horizontal reaction force).

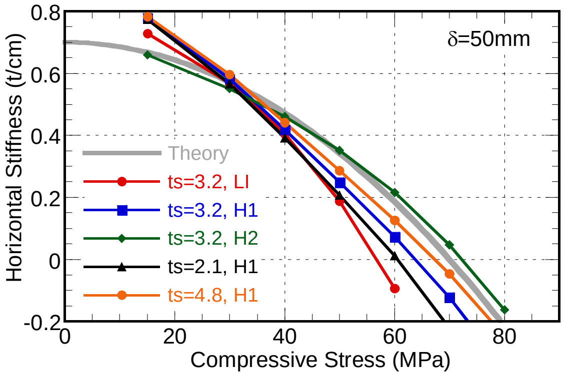


Fig.4 Relationship to Horizontal Stiffness of Compressive Stress

In Fig.5 (a), the compressive stiffness lowers as the horizontal deformation increases. It is considered that most of the compressive load is supported by the wrapped parts on the top and bottom of the bearings when the bearing is deformed under shear stress. In this case, the compressive stiffness is considered to be proportional to the wrapped area, and the area is obtained by approximate calculation with Equation (4).

$$\frac{K_V}{K_{V0}} = \frac{A_e}{A} = 1 - 1.2\frac{\delta}{D} \tag{4}$$

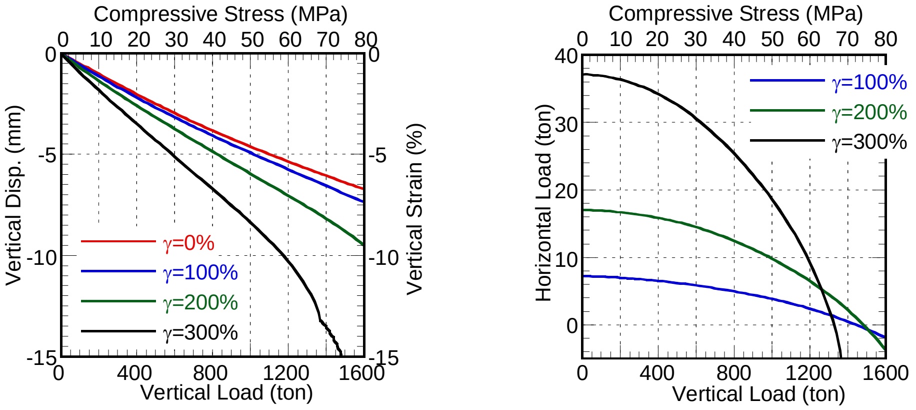
where, K_V : the compressive stiffness with shear deformation

K_{V0} : the compressive stiffness without shear deformation

A_e : the wrapped area of the top and bottom of a bearing,

A : the cross sectional area of a rubber bearing

δ : horizontal deformation, D: the diameter of a rubber bearing



(a) ts=3.2mm, H2-type

(b) ts=3.2mm, H2-type

Fig.5 Vertical Characteristics of Compression Loading under Constant Shear Deformation

Table 3 shows the comparison of the compressive stiffness of each analytical model. The compressive stiffness is the secant stiffness when the compressive load is 400tons (about 20MPa). The compressive stiffness varied slightly according to the shear modulus, although the margin is small and they are almost the same. The ratios of shear deformation from 0mm to the compressive stiffness are also shown in the brackets in Table 3. When they are compared to each other, the decrease in the compressive stiffness of Type H2 rubber material is the smallest.

The theoretical values obtained by Equation (4) predict the smallest decrease in the compressive stiffness compared to the analytical results. This is because the effect of strain hardening of rubber material becomes significant when the shear deformation is large. Also, it is not only the top and bottom faces that support the compressive load.

Table 3 Compressive Stiffness at Vertical Load 400ton (t/cm)

δ (mm)	ts=3.2mm			ts=2.1mm	ts=4.8mm	Theory Eq.(4)
	LI	H1	H2	H1	H1	
0	2147 (1.0)	2347 (1.0)	1973 (1.0)	2340 (1.0)	2348 (1.0)	(1.00)
100	1904 (0.887)	2007 (0.855)	1824 (0.924)	1986 (0.849)	2022 (0.861)	(0.76)
200	1403 (0.653)	1531 (0.652)	1544 (0.783)	1500 (0.641)	1553 (0.661)	(0.52)
300	866 (0.403)	1094 (0.466)	1145 (0.580)	1053 (0.450)	1130 (0.481)	(0.28)

Fig. 5 (b) shows that the horizontal load (reaction force) decreases as the compressive load is increased. The state in which the horizontal reaction force becomes zero means a loss of horizontal resistance and causes the rubber bearing to become unstable. The compressive load when the horizontal reaction force is zero is called ultimate load.

The analytical results for all of the models are shown in Table 4. The ultimate loads of the different types of rubber material, from highest to lowest, are Type H2, followed by HI and LI. It is obvious that the stability of the bearing under the ultimate load is related not only to the initial shear modulus but also to the stress-strain relation, or the degree of hardening of the rubber material. It is also obvious that the ultimate load is higher as the interlayer steel plate is thicker.

Table 4 Ultimate Stress (MPa)

ts	3.2mm			2.1mm	4.8mm
δ (mm)	LI	H1	H2	H1	H1
100	58.0	65.0	72.0	61.0	68.0
200	53.5	65.0	74.0	60.5	67.6
300	45.9	55.0	66.4	50.2	58.1

The relationship between the ultimate load and the shear strain is shown in Fig. 6. In the figure, the buckling points obtained from the hysterisis loops in Fig. 3 are also shown, drawn as big marks. If the amount of horizontal deformation and compressive load were drawn when the horizontal load is zero on the hysterisis curve in Fig. 3, the results would be almost the same as the ultimate load points (small marks) in Fig. 6.

Fig. 6 shows that the value of the ultimate load can be higher than the buckling point. In particular, Type LI and H1 rubber material have a strong tendency towards this. In order to find the buckling points and stable deformation capacity of rubber bearings, the shear loading test under a compressive load is necessary. However, there is no big difference in the values of the buckling and ultimate points of Type H2. This is because the hysteresis loop of the rubber bearing is more stable with the stronger hardening property of rubber material. As a result, the buckling load of the bearing is high.

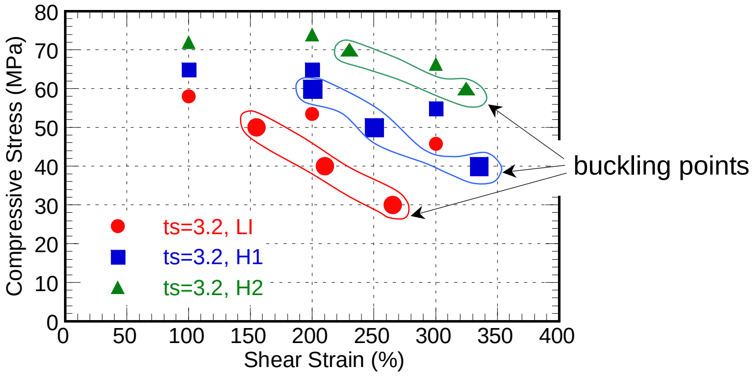
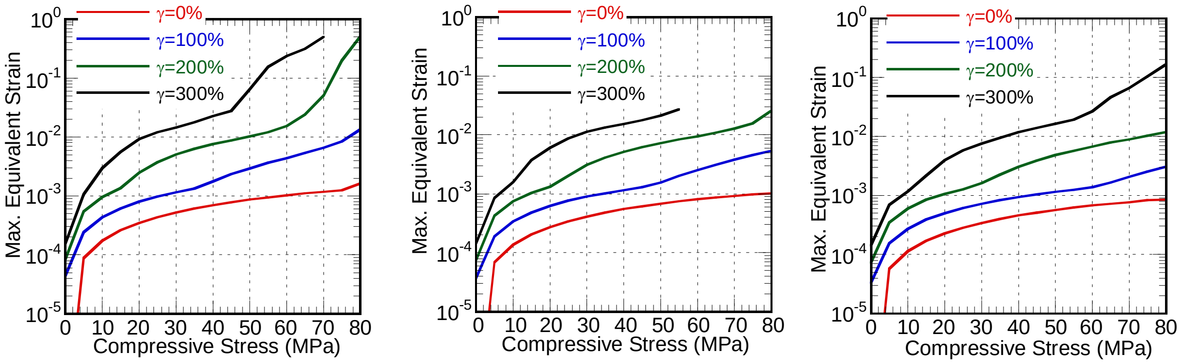


Fig.6 Relationship between Compressive Stress and Shear Strain at Unstable Condition

Fig. 7 shows the relationship between the maximum equivalent strain and compressive stress. The yield strain of the interlayer steel plate is 1.4×10^{-3}, and a part of the plate becomes plastic when the yield strain exceeds the value. When the interlayer steel plate is thick, the strain of the plate lower, and the area of the plasticity is small.

Fig.8 shows the equivalent stress of the fourth interlayer steel plate from the top when the shear deformation of the rubber bearing is 200mm (about 200% shear strain). The interlayer steel plate is 3.2mm thick, and the deformation is magnified to 20 times. When the compressive load is changed from 30MPa to 60MPa, the plasticity increases and the out-of-plane deformation becomes larger. It has been found that the concentration of plasticity is around the center hole when it is made in the analytical model ([1][2]). It is feared that making a center hole in a rubber bearing would cause the bearing to become more plastic and therefore decrease the ultimate performance.



(a) ts=2.1mm, H1-type (b) ts=3.2mm, H1-type (c) ts=4.8mm, H1-type

Fig.7 Maximum Equivalent Strain of Interlayer Steel Plates

4. Conclusion

Natural rubber bearings were examined by the finite element analysis method and the findings are as follows:

1) The properties of the rubber material of the rubber bearings, especially the hardening properties, have a significant effect on the ultimate load and buckling load of rubber

bearings. In order to estimate the ultimate performance of rubber bearings, not only the shear modulus but also hardening behavior of the rubber material should be taken into consideration.

2) As the interlayer steel plate is thicker, the ultimate load is higher and the rubber bearing will be more stable. The interlayer steel plates are likely to become plastic, and it is necessary to examine how the plasticity affects the performance of the rubber bearing.

3) In order to determine the ultimate performance of rubber bearings, a shear deformation test should be conducted on bearings under a constant compressive load. It will lead to an overestimation of the ultimate performance of the bearings when a compressive load is applied on the bearings under a constant horizontal deformation.

4) In this study, only analytical results were examined. The task ahead is to conduct an experiment and compare the experimental results to the analytical results and propose equations in order to estimate the buckling point of rubber bearings.

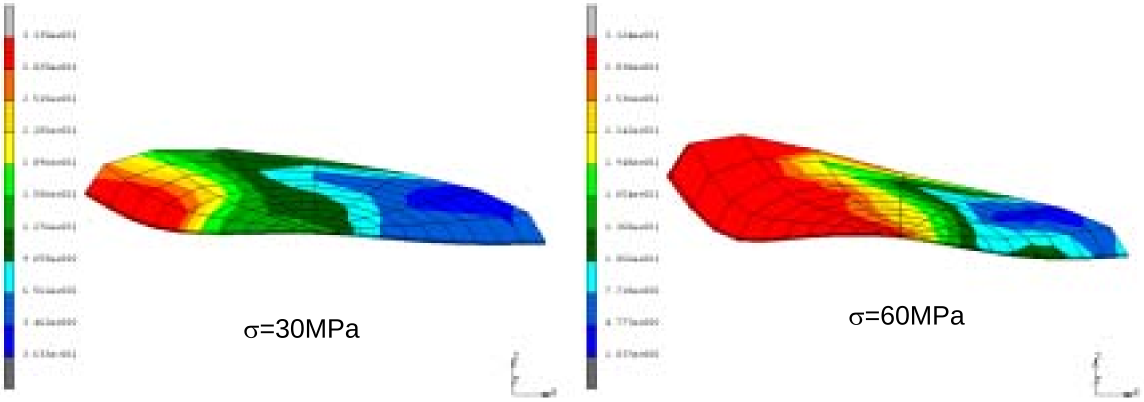


Fig.8 Distribution of Equivalent Stress of Interlayer Steel Plate (4th layer)

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